On the modelling of effects of negative production of temperature-fluctuation intensity in the turbulent mixing layer

By A. F. KURBATSKII and N. N. YANENKO

Institute of Theoretical and Applied Mechanics, U.S.S.R. Academy of Sciences,
Novosibirsk 630090, U.S.S.R.

Numerical results are presented for the modelling of the spread of heat as a passive scalar contaminant on the basis of a second-order closure model in the mixing layer with an asymmetric mean-temperature profile superimposed on it. Present calculations are in reasonable agreement with experimental data on the region of countergradient transport for heat where the direction of heat diffusion is opposite to the mean gradient diffusion and where the production of temperature fluctuation intensity is negative.

1. Introduction

In a series of experimental studies of classical flows of the mechanics of turbulence like jets, wakes, mixing layers, channel flows (Batchelor 1950; Fabris 1979; Beguier, Fulachier & Keffer 1978*a*; Beguier *et al.* 1978*b*) the violation of local phenomenological relations is found for turbulent flows of the gradient-diffusion type (relations of the Boussinesq type), i.e. those in the form of gradients of mean quantities. The points at which the turbulent flux (e.g. $-\langle uv \rangle = 0$ or $-\langle \theta v \rangle = 0$ in the case of planar flow) and the mean gradient ($\partial U/\partial y = 0$ or $\partial \Theta/\partial y = 0$ respectively) vanish prove to be displaced from one another. The so-called displacement zone is formed. Within this zone the turbulent flux and the mean gradient have the same sign. This means that within the limits of the displacement zone the heat (or impulse) transfer occurs in the gradient direction (that is in the direction opposite to the transfer direction due to the mean gradient). From the point of view of the gradient-transfer mechanism in the spirit of the Boussinesq phenomenological law this situation would correspond to a negative coefficient of turbulent transport.

The zone of displacement arises in flows with asymmetrical mean profiles (Beguier *et al.* 1978*b*) as well as in the flows with symmetric mean profiles (Batchelor 1950; Fabris 1979) of some statistical properties of the turbulent flow. According to direct measurements (Beguier *et al.* 1978*a*), the turbulence production ($\mathscr{P} \equiv -\langle uv \rangle \, \partial U/\partial y$) or the production of the intensity of temperature-field fluctuations ($\mathscr{P}_{\theta v} \equiv -\langle \theta v \rangle \, \partial \Theta/\partial y$) is negative in the zones of displacement. In the thermally non-uniform turbulent mixing layer with an asymmetrical mean-temperature profile in whose zone of displacement the production of the intensity of temperature fluctuations proves to be negative, the temperature could be considered as a passive contaminant (due to a small absoute difference in temperatures that took place in experiments). Therefore the temperature did not influence substantially the flow dynamics, and, in particular, the production of the turbulence energy. The negative production of the intensity of temperature fluctuations in such a mixing layer is a consequence of

the dynamics of the eddy flow structure. Only the asymmetrical transport of flow properties determined by large-scale eddies seems to cause the abovementioned anomalous phenomena within the zone of displacement. The spectral analysis of the displacement zone has shown (Beguier *et al.* 1978*b*) that the cospectrum of the transverse component of the turbulent heat flux can be conventionally subdivided into two parts. The cospectrum part corresponding to high frequencies (small-scale eddies) can be kept within the limits of the classical scheme of transport after the mechanism of gradient diffusion, whereas the low-cospectrum frequencies (large-scale edies) cause the heat transfer in the direction of the mean temperature gradient. This transfer (opposite sign) just represents the basic mechanism of the negative production. The presence of the two domains in the cospectrum seems to confirm the assumption of Townsend (1956) on the bimodal structure of the turbulent transfer mechanism. According to this assumption the turbulent flux (of the momentum, enthalpy etc.) can be roughly represented in the form of a sum of two parts. The part corresponding to the transport by small-scale eddies is described by local relationships of the Boussinesq type, and for the other part responsible for the transport by large-scale eddies it was proposed by Townsend (1956) to introduce the notion of 'bulk convection'. However, the model representation of the 'bulk convection' (cf. e.g. Batchelor 1950; Beguier *et al.* 1978*b*) reduces in the process of its practical realization to the assumption on the smallness of the displacement zone, requires additional empirical information, and involves consideration of additional parameters of the turbulent flow (incompleteness of the expression obtained for the turbulent flux). This notion makes it impossible to obtain a realizable model for the investigation of the non-local effects due to turbulent transport in the numerical simulations of the development of various turbulent flows.

All the phenomenological relationships of the gradient-diffusion type employed for the modelling of the mechanism of turbulent transport have in their physical background an analogy with the kinetic theory of gases. Carrying through this analogy further, it can be thought that the turbulent transport by large-scale eddies which is responsible, in particular, for the development of local zones of negative production (in asymmetrical flows), is from the point of view of the kinetic theory the transport at large free-path length. In this case the Fourier law for the heat flux and the rheological Stokes equation of state for the friction stress do not hold for the limit laws at small free-path length. In the kinetic theory of gases the stress tensor is a dependent variable in the descriptions of the transport at large free-path length (Grad 1949). This variable satisfies a certain differential conservation equation. The heat flux is also a dependent variable. This approach to the description of turbulent transport has already been developed to a certain extent, and has been successfully applied also to the computation of turbulent flows in the presence of displacement zones (Hanjalic & Launder 1972; Kurbatskii & Onufriev 1979). Therefore, if an analogy with the kinetic theory of transport at large free-path length is carried through, it can be assumed that also in the phenomenological theory of turbulent transport the computation of, for example, tensor components of Reynolds stresses and the components of the turbulent heat-flux vector from corresponding differential transport equations (having the meaning of conservation laws) is a description of the transport at a large scale of correlation in turbulent flow. And in this sense such a description takes into account the non-local character of the turbulent-transport mechanism. A corroboration of this follows from the comparison presented herein of the experimental data of Beguier *et al.* (1978*b*) with the results of the numerical modelling of turbulent transport in the mixing layer with an asymmetrical mean-

temperature profile on the basis of an approximate system of differential transport equations including the equations for the components of the Reynolds stress tensor and for the components of the turbulent heat-flux vector. This comparison shows that in accordance with the experimental data a displacement zone is found which has within its limits a negative production of the intensity of turbulent temperature fluctuations and which has turbulent enthalpy transport in the direction opposite to that of the motion due to the mean-temperature gradient.

2. Turbulent-transport equations

An approximate equation system for the turbulent transport of statistical properties of the velocity field and of the scalar field (temperature or concentration) is presented in Kurbatskii (1975). Therefore the corresponding equations are written in what follows without explanations. Only some notations will be changed; they will be specified.

As in Beguier *et al.* (1978*a*) we consider the development of a plane turbulent flow – the mixing layer arising at the edge of a slightly heated plane jet discharged into the surroundings on the one hand and a uniform flow moving at a speed equal to the jet speed on the other hand (figure 1). The absolute difference between the temperatures of the jet and the ambient gas was insignificant in the experiments of Beguier *et al.* (1978*b*). As a result of this the temperature could be considered as a passive 'contaminant', as had already been mentioned above (the buoyancy did not influence the motion of the medium substantially), and the mean temperature distribution superimposed upon the velocity field in this flow had an asymmetrical character.

The equation system for the moments of the velocity field in the approximation of a thin shear layer for a plane developed free turbulent flow, neglecting the effects of molecular viscosity, has the form

$$
\left.
\begin{aligned}
&U\frac{\partial U}{\partial x} + \tilde{V}\frac{\partial U}{\partial y} + \frac{\partial \langle uv \rangle}{\partial y} = 0, \\
&U\frac{\partial \langle uv \rangle}{\partial x} + \tilde{V}\frac{\partial \langle uv \rangle}{\partial y} - \mu_1\frac{\partial}{\partial y}\left\{\tau\left(\langle uv \rangle \frac{\partial \langle v^2 \rangle}{\partial y} + 2\langle v^2 \rangle\frac{\partial \langle uv \rangle}{\partial y}\right)\right\} \\
&\hspace{6cm} + \langle v^2 \rangle\frac{\partial U}{\partial y} + \frac{2\nu_1 + a}{\tau}\langle uv \rangle = 0, \\
&U\frac{\partial \langle v^2 \rangle}{\partial x} + \tilde{V}\frac{\partial \langle v^2 \rangle}{\partial y} - \mu_1\frac{\partial}{\partial y}\left\{3\tau\langle v^2 \rangle\frac{\partial \langle v^2 \rangle}{\partial y}\right\} + \frac{a}{\tau}\langle v^2 \rangle + \frac{2\nu_1}{\tau}[\langle v^2 \rangle - \tfrac{2}{3}E] = 0, \\
&U\frac{\partial E}{\partial x} + \tilde{V}\frac{\partial E}{\partial y} - \mu_1\frac{\partial}{\partial y}\left\{\tau\left(\langle v^2 \rangle\frac{\partial E}{\partial y} + \langle uv \rangle\frac{\partial \langle uv \rangle}{\partial y} + \langle v^2 \rangle\frac{\partial \langle v^2 \rangle}{\partial y}\right)\right\} + \langle uv \rangle\frac{\partial U}{\partial y} + \frac{a}{\tau}E = 0.
\end{aligned}
\right\} \quad (2.1)
$$

In the system (2.1) U is the mean longitudinal velocity, $\tilde{V}$ is the mean (mass-weighted) transverse flow velocity, $-\langle uv \rangle$ the Reynolds stress, $\tau = ALE^{-\frac{1}{2}}$ is the specific timescale for the turbulent motion, $E = \frac{1}{2}(\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle)$ is the mean density (per unit mass) of the kinetic energy of turbulence, u, v, w are turbulent velocity fluctuations to the mixing layer (figure 1); L is the integral turbulence scale, $A \approx 4$, $\nu_1 \approx 0.5$, $a \approx 1$ are empirical constants whose numerical values are found from the solution of some simplest problems of the turbulent transport (Kurbatskii 1975). In the domain of self-similarity of statistical characteristics of the turbulence of the mixing layer, the longitudinal component of the mean momentum can be approximated
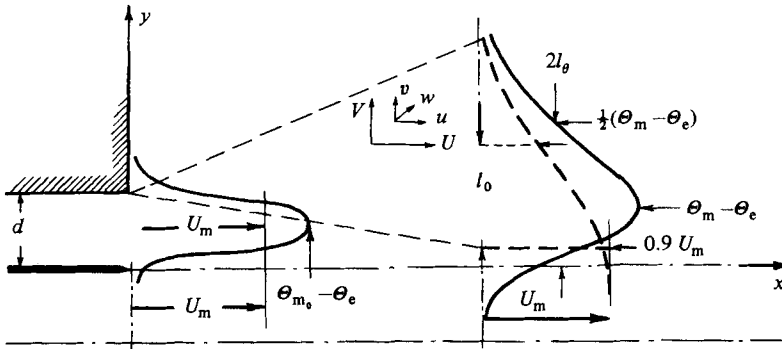
FIGURE 1. Definition sketch of flow field.

by an expression $\langle \rho u \rangle \approx \langle \rho \rangle U$ with an error not exceeding 0.5 % (Beguier *et al.* 1978*b*). The angular brackets mean the statistical averaging, and $\langle \rho \rangle$ is the constant magnitude of density.

For the transverse component of the mean momentum the mass-weighted expression:

$$\tilde{V} = \frac{\langle \rho v \rangle}{\langle \rho \rangle} = V - \frac{\langle \theta v \rangle}{\Theta} \tag{2.2}$$

should be taken, because the second item on the right-hand side of (2.2) is not always negligibly small. In the expression (2.2) Θ is the mean temperature, $\langle \theta v \rangle$ is the transverse component of the turbulent heat flux, V is the mean value of the transverse component of the flow velocity. In accordance with Beguier *et al.* (1978*b*) the approximations of the turbulent heat flux $\langle \rho v \theta \rangle \approx \langle \rho \rangle \langle \theta v \rangle$ and of the turbulent momentum flux $\langle \rho u v \rangle \approx \langle \rho \rangle \langle uv \rangle$ have an error that does not exceed 1 %. The system of equations (2.1) for the turbulent transport is written in a complete form (with respect to the moments); the processes of turbulent diffusion (the third one-point moments) appearing in the equations for the second moments of the system (2.1) are replaced, approximately, by the second moments employing the hypothesis of fourth moments in the way indicated by, for example, Hanjalic & Launder (1972). That is, by using the following anisotropic relationship for the third moments:

$$\langle u_\alpha u_\beta u_\gamma \rangle = -\mu_1 \tau \left[\langle u_\alpha u_\kappa \rangle \frac{\partial \langle u_\beta u_\gamma \rangle}{\partial x_\kappa} + \langle u_\beta u_\kappa \rangle \frac{\partial \langle u_\alpha u_\gamma \rangle}{\partial x_\kappa} + \langle u_\gamma u_\kappa \rangle \frac{\partial \langle u_\alpha u_\beta \rangle}{\partial x_\kappa} \right], \tag{2.3}$$

where μ_1 is a numerical constant whose value was taken to be equal to 0.11. This value was found by numerical optimization, and was employed in various stimulations of turbulent transport. Since the equation for mass onservation has the form $\partial U/\partial x + \partial \tilde{V}/\partial y = 0$ for a plane mixing layer, it appeared to be convenient to find the mean-weighted transverse flow velocity $\tilde{V}$ from the expression

$$\tilde{V} = -\int_{-\infty}^{y} \frac{\partial U}{\partial x} \, dy. \tag{2.4}$$

The velocity U_m of the uniform flow at its downstream boundary was taken as a scaling velocity for the flow in the mixing layer. The expression for the turbulence scale L (the correlation scale in whose limits the element of the fluid makes a correlated fluctuation displacement in the medium) was taken in the algebraic form consistent with the self-similar character of its change in the downstream direction:

$L \sim x - x_0$. This law of the change in the scale in the far domain of the mixing layer corresponds to the one observed experimentally (x_0 is the virtual origin of the mixing layer). Only the quantity for the turbulence scale is employed; the intermittence in the flow is not taken into account. For convenience of comparisons with experimental data, the expression for the scale L was determined by some specific conventional width l_0 of the mixing layer which was taken as a distance between the points $y = y_{0.5}$ and $y = y_{0.9}$ (figure 1) at which $U = 0.5U_m$ and $U = 0.9U_m$ respectively: $L = \kappa l_0$ (where $\kappa \approx 0.5$ – an empirical constant). The system (2.1) is written in a non-dimensional form by using U_m as a velocity scale, and as a characteristic lengthscale the height d of the slot from which the heated jet was discharged was used. In comparisons of the computed distributions of the mean quantities and of the moments with experimental data, the self-similar variable $\eta = (y - y_0)/l_0$ was employed.

The equations of the turbulent transport of statistical properties of the temperature field in the same approximation of a thin shear layer neglecting the effects of molecular heat conduction have the form (Kurbatskii 1975):

$$
\left.
\begin{aligned}
&U\frac{\partial \Theta}{\partial x} + \tilde{V}\frac{\partial \Theta}{\partial y} + \frac{\partial \langle \theta v \rangle}{\partial y} = 0, \\
&U\frac{\partial \langle \theta v \rangle}{\partial x} + \tilde{V}\frac{\partial \langle \theta v \rangle}{\partial y} - \mu_2 \frac{\partial}{\partial y}\left\{\tau\left(2\langle v^2 \rangle \frac{\partial \langle \theta v \rangle}{\partial y} + \langle \theta v \rangle \frac{\partial \langle v^2 \rangle}{\partial y}\right)\right\} \\
&\hspace{3cm} + \langle v^2 \rangle \frac{\partial \Theta}{\partial y} + (\nu_1 + \nu_2 + 3b_0 A)\frac{\langle \theta v \rangle}{\tau} = 0;
\end{aligned}
\right\} \quad (2.5)
$$

$$
\left.
\begin{aligned}
&U\frac{\partial \langle \theta^2 \rangle}{\partial x} + \tilde{V}\frac{\partial \langle \theta^2 \rangle}{\partial x} + \frac{\partial \langle \theta^2 v \rangle}{\partial y} + 2\langle \theta v \rangle \frac{\partial \Theta}{\partial y} + \frac{6b_0 A}{\tau}\langle \theta^2 \rangle = 0, \\
&U\frac{\partial \langle \theta^2 v \rangle}{\partial x} + \tilde{V}\frac{\partial \langle \theta^2 v \rangle}{\partial y} + 2\langle \theta v^2 \rangle \frac{\partial \Theta}{\partial y} + \langle v^2 \rangle \frac{\partial \langle \theta^2 \rangle}{\partial y} \\
&\hspace{3cm} + 2\langle \theta v \rangle \frac{\partial \langle \theta v \rangle}{\partial y} + (\nu_1 + 2\nu_2 + 6b_0 A)\frac{\langle \theta^2 v \rangle}{\tau} = 0.
\end{aligned}
\right\} \quad (2.6)
$$

In (2.5), (2.6) Θ is the mean temperature in the mixing layer, b_0 is an empirical constant in the term describing the scalar dissipation of the intensity of turbulent temperature fluctuations $\langle \theta^2 \rangle$. The numerical value of the constant $b_0 \approx 0.042$ was found from the solution of a problem on the determination of the distribution of the temperature-fluctuation intensity in the thermal turbulent mixing layer formed behind a partially heated lattice in the uniform turbulent flow. The value of the constant ν_2 is taken to be equal to 0.5. The difference $\Theta_m - \Theta_e$ between the maximum temperature in the mixing layer and the ambient temperature (figure 1) was taken as the mean temperature scale. For comparisons with experimental data of Beguier *et al.* (1978*b*) the mean quantities and the moments of the temperature field found from the solution of the systems (2.5), (2.6) were represented by using a self-similar coordinate $\eta_\theta = (y - y_0)/l_\theta$. In this expression y_0 is the coordinate of the maximum in the mean-temperature profile, and l_θ is the scale characterizing the width of the mean-temperature distribution in some section of the mixing layer: the distance between the points at which the mean-temperature value is equal to half its maximum value is taken to be equal to $2l_\theta$ (figure 1).

As for the velocity field, the complete form of the equation system (2.5) has been obtained by using an approximate anisotropic closure relationship for the third mixed

moment of the velocity and of the temperature and by using the generalized hypothesis on the fourth moments:

$$\langle u_\alpha u_\beta \theta \rangle = -\mu_2 \tau \left[\langle u_\kappa u_\alpha \rangle \frac{\partial \langle u_\beta \theta \rangle}{\partial x_\kappa} + \langle u_\kappa u_\beta \rangle \frac{\partial \langle u_\alpha \theta \rangle}{\partial x_\kappa} + \langle u_\kappa \theta \rangle \frac{\partial \langle u_\alpha u_\beta \rangle}{\partial x_\kappa} \right], \quad (2.7)$$

where μ_2 is a constant to be found by the numerical optimization. Its numerical value was taken to be equal to 0.15. The closure relationship for the process of diffusion of the transverse component of the turbulent heat flux following from (2.7) has the form

$$\langle \theta v^2 \rangle = -\mu_2 \tau \left[2 \langle v^2 \rangle \frac{\partial \langle \theta v \rangle}{\partial y} + \langle \theta v \rangle \frac{\partial \langle v^2 \rangle}{\partial y} \right]. \quad (2.8)$$

The relation (2.8) is employed in the second equation of the system (2.5) to obtain its closed form. The closed form of the system (2.6) is obtained by using the generalized hypothesis of fourth moments:

$$\langle u_\kappa u_\alpha \theta^2 \rangle = \langle u_\kappa u_\alpha \rangle \langle \theta^2 \rangle + 2 \langle u_\kappa \theta \rangle \langle u_\alpha \theta \rangle. \quad (2.9)$$

That is, the fourth moment $\langle u_\kappa u_\alpha \theta \theta \rangle$ in the equation for the process of turbulent diffusion of the temperature-fluctuation intensity, of magnitude $\langle \theta^2 u_\alpha \rangle$, is replaced by the second moments with the aid of the relation (2.9). Thus the statistical characteristics of the temperature field in the mixing layer are found by the successive solution of two complete (uncoupled) equation systems (2.5) and (2.6) for Θ, $\langle \theta v \rangle$, and $\langle \theta^2 \rangle$, $\langle \theta^2 v \rangle$ respectively.

We now make a few remarks about the quasinormality hypothesis (the hypothesis of fourth moments, or Millionshchikov's hypothesis). At present there are no accurate and detailed experimental data on the initial distributons of third moments. As a consequence of this, full and final conclusions regarding Millionshchikov's hypothesis cannot be drawn. Nevertheless, the critical analysis (Orszag 1970) of the quasinormal closure of the equation system for moments in the case of homogeneous and isotropic turbulence leads to a conclusion that in this case the quasinormality hypothesis does not ensure a sufficient decay (or weakening) of the eddies by virtue of their nonlinear interactions: the spectral transport proves to be reversible everywhere and leads to an oscillatory behaviour, and this is displayed in the excessive generation of the third moments and the appearance of negative values of the spectral density of the turbulence energy as the time increases. However, there remains the fact that the application of the quasinormality hypothesis in various problems of turbulent transport in flows with a permanent source of turbulence energy production (by virtue of the shear and the buoyancy), alongside accurate approximations of correlations with pressure fluctuations (e.g. in the equations for the third moments having (Kurbatskii & Onufriev 1979) the relaxation form possessing the property of an irreversible decay and eliminating the abovementioned oscillatory behaviour) guarantees that the third moments will not exceed their physically admissible values. Of course, the assertion stated above has a heuristic character, but the results of computations carried through (cf. e.g. Hanjalic & Launder 1972; Kurbatskii & Onufriev 1979; Kurbatskii 1975) seem to confirm it.

An interesting feature of the equations (2.5) and (2.6) for the moments of the temperature field is the fact that the system (2.6) is of a completely hyperbolic type (this is a quasilinear system of first-order partial differential equations being hyperbolic after Petrovskii). The system has two real characteristic roots

$$\lambda_{1,2} (\equiv \mathrm{d}x : \mathrm{d}y) = \frac{U(\bar{V} \pm \langle v^2 \rangle^{\frac{1}{2}})}{\bar{V}^2 - \langle v^2 \rangle},$$

although it contains in itself a physically irreversible mechanism of scalar dissipation for the functions $\langle\theta^2\rangle$ and $\langle\theta^2 v\rangle$. The system (2.5) has a double 'boundary-layer' characteristic $\lambda_{1,2} = 0$ ($x = $ const), generated by an equation of parabolic type; the turbulent heat flux and one real (different from the trivial one) characteristic $\lambda_3 = U/\tilde{V}$ ($\mathrm{d}x{:}\mathrm{d}y = U/\tilde{V}$) coinciding with the streamline and generated by the equation for the mean temperature Θ. Owing to the presence of a real characteristic λ_3, the system (2.5) cannot be related to the parabolic type. The same is also valid for the system of equations (2.1) for the moments of the velocity field, which has the streamline as one characteristic and the triple 'boundary-layer' characteristic.

For the numerical integration of the equation systems (2.1), (2.5) and (2.6) the method of finite differences has been employed. By virtue of the abovementioned mathematical properties of the equations, the development of an efficient numerical algorithm represents, generally speaking, an independent and complicated problem. However, the main purpose of the present work was to elucidate the capacity for 'work' of the transport equations (2.5), (2.6) for the modelling of the non-local mechanism of turbulent transport (in asymmetrical free turbulent flows). Therefore we used the simplest first-order (of course, not the most efficient) implicit-difference scheme possessing the property of absolute computational stability (the latter property has been established experimentally). The details of corresponding finite-difference approximations of the second-order differential operators of the equations (2.1), (2.5) and (2.6) can be found in Kurbatskii (1975). The difference scheme is effectively realized by a vector recurrence method (on the rectangular Eulerian grid). The accurate (experimental) initial data for the mean quantities and moments sought for are unknown. In the process of numerical solution the parameter distributions, being close in some sense to the experimental ones, were set as the initial data. This circumstance did not represent any serious difficulty for the development of the mixing layer considered here, because in the experiments of Beguier *et al.* (1978*b*) a self-similar character of the change in basic statistical characteristics of the turbulence field in the far region of the mixing layer (at $x/d \geqslant 10$) had been established.

3. Numerical results

Numerical results of the modelling of turbulent momentum and heat transport in the mixing layer with an asymmetrical mean-temperature profile are presented in the form of graphs.

As has already been noted, the zone of displacement is formed as a result of the mutual transverse displacement of the points at which the turbulent flux and the mean gradient vanish. A comparatively small width of this zone (about 15 % of the conventional width $2l_\theta$) imposes corresponding restrictions on the accuracy of numerical computations and, consequently, on the ability of finding the zone of displacement adequate to explain the experimental data. These restrictions proved to be rather rigid for the simple difference scheme employed; the distance along the longitudinal coordinate x that was reached in the numerical solution corresponded to the section having $x/d \approx 15$. The advancement to this section in the numerical solution could be identified, for example, by the magnitude of the characteristic transverse scale of the temperature field (in the section $x/d \approx 15$ the scale $l_\theta \approx 1.1$ (cf. figure 7 of Beguier *et al.* 1978*a*). The change in the scales l_0 and l_θ in computations was linear with respect to the longitudinal coordinate, as in the experiments of Beguier *et al.* (1978*b*).

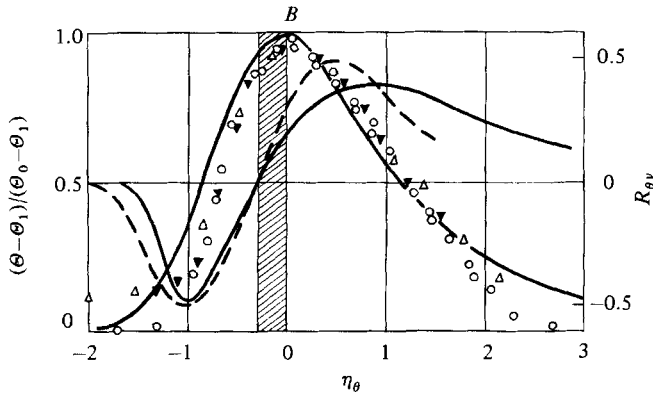The mean-temperature profile in the initial section was set to be close to the

FIGURE 2. Self-similar profiles of mean temperature and velocity and temperature correlation $R_{\theta v}$. Experimental data from Beguier *et al.* (1978b): ---, $x/d = 30$; $\triangle$, 10; $\blacktriangledown$, 20; $\bigcirc$, 30. Calculation: ——.
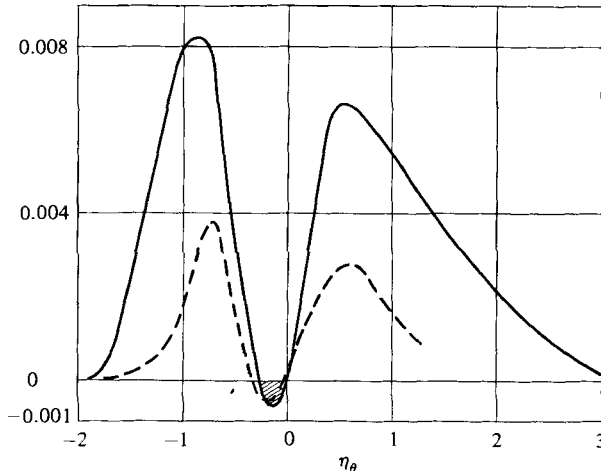

FIGURE 3. Production of temperature-fluctuation intensity

$$\mathscr{P}_{\theta v} = -[d/(\Theta_\mathrm{m} - \Theta_\mathrm{e})^2 \, U_\mathrm{m}] \, [\langle \theta v \rangle \, \partial\Theta/\partial y].$$

Experimental data from Beguier *et al.* (1978b): ---, $x/d = 30$. Calculation: ——.

experimental one (cf. figure 3 of Beguier *et al.* 1978b); initial profiles of the second and third moments were set by using local equilibrium relationships (that is under the assumption on the balance between the production and dissipation).

The measurements of statistical characteristics of the temperature field (in particular, the correlation coefficient $R_{\theta v} = \langle \theta v \rangle / \langle \theta^2 \rangle^{\frac{1}{2}} \langle v^2 \rangle^{\frac{1}{2}}$, the value of the temperature-fluctuation intensity $\langle \theta^2 \rangle$, as well as the production of temperature-fluctuation intensity $\mathscr{P}_{\theta v} (\equiv -\langle \theta v \rangle (\partial\Theta/\partial y))$ have been carried out in these experiments in one section, $x/d = 30$, (with the exception of the mean temperature profile, measured in some downstream sections).

The numerical results in figures 2–4 (solid lines) show that physical effects of the non-local character of the turbulent transport mechanism – contragradient property of the heat flux $\langle \theta v \rangle$ and negativeness of the production of temperature-fluctuation intensity $\mathscr{P}_{\theta v}$ – are forecast numerically by the turbulent-transport model and that the numerical results agree with experimental data (the points of different configu-
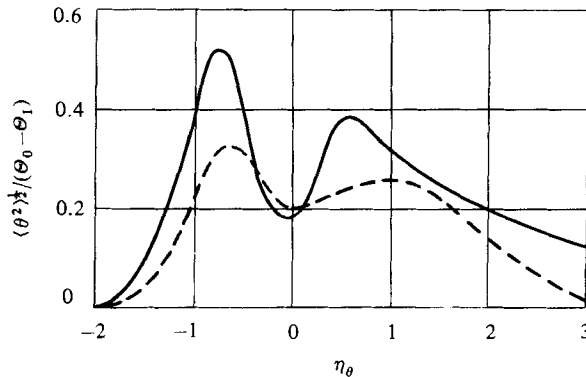
FIGURE 4. Profile of temperature-fluctuation intensity. Experimental data from
Beguier *et al.* (1978*b*): ---, $x/d = 30$. Calculation: ——.

ration and dashed lines). However, there are quantitative discrepancies between
calculations and experimental data. One of the reasons for these discrepancies is
explained by the fact that the numerical results for the second moments of the random
temperature field $R_{\theta v}$, $\langle \theta^2 \rangle$ as well as the values of production $\mathscr{P}_{\theta v}$ and experimental
distributions of these quantities presented in figures 2–4 refer to different cross-sections
of the mixing layer.

Figure 2 shows the mean-temperature distribution across the mixing layer. Within
the limits of the errors in positions of experimental points it can be thought that an
approximately self-similar character of the change in the mean-temperature profile
in experiments had been established behind the section $x/d \geqslant 10$. The computed
mean-temperature profile has an asymmetrical form: the gradient is slightly steeper
on the high-velocity side of the mixing layer ($\eta_\theta < 0$). The distribution of the
correlation coefficient $R_{\theta v}$ across the mixing layer is also depicted in figure 2. It can
be seen that the width of the displacement zone obtained in computations agrees
with the experimental one, thus showing the good resolving power of both the
turbulent-transport model and the numerical method of its realization. Figure 3
presents the second basic result of the present work: the distribution of the transverse
component of the production of temperature turbulent-fluctuation intensity
$\mathscr{P}_{\theta v} \equiv -\langle \theta v \rangle \, \partial\Theta/\partial y$ across the mixing layer. The main thing to which attention
should be drawn in this figure is a negative-production zone of the calculated quantity
$\mathscr{P}_{\theta v}$ which agrees with the process of its formation measured directly in the
experiments of Beguier *et al.* (1978*b*). And, finally, figure 4 shows a distribution across
the mixing layer of the magnitude of mean-square turbulent fluctuations of the
temperature. Note that the use of the system of two transport equations for the second
moment $\langle \theta^2 \rangle$ and the third moment $\langle \theta^2 v \rangle$ enabled us to obtain the correct form of
the distribution $\langle \theta^2 \rangle^{\frac{1}{2}}$ curve having two specific maxima. Calculation data on
statistical characteristics of the random velocity field in the mixing layer described
by the system (2.1) of equations of turbulent transport are not presented here. These
data are available in Kurbatskii (1975).

4. Concluding remarks

The general conclusion following from the present work is that, in the turbulent-
transport phenomena where the diffusion processes of certain statistical character-

istics of random fields of velocity and temperature play a determining role for the dynamics in the whole flow field (e.g. the turbulence energy transport across the layer mixed by a fluctuating buoyancy force is contragradient over the whole layer width; cf. Willis & Deardorf 1974) or in its part (relatively small part of the flow field – the displacement zone AB in figure 2), the use of the differential equations of turbulent transport for the moments of second and third orders (see e.g. Kurbatskii 1979) enable one to describe the non-local character of the turbulent transport mechanism within the framework of the model realized. The recent results of Dekeyser (1982) on the modelling of turbulent heat transport in the plane jet with asymmetric mean-temperature profile are also in agreement with this conclusion.

REFERENCES

BATCHELOR, G. K. 1950 Note on free turbulent flows with special reference to the two-dimensional wake. *J. Aero. Sci.* **17**, 441–445.

BEGUIER, C., FULACHIER, L. & KEFFER, J. F. 1978*a* The turbulent mixing layer with an asymmetrical distribution of temperature. *J. Fluid Mech.* **89**, 561–587.

BEGUIER, C., GIRALT, F., FULACHIER, L. & KEFFER, J. F. 1978*b* Negative production in turbulent shear flows. In *Structure and Mechanisms of Turbulence II* (ed. H. Fiedler). Lecture Notes in Physics, vol. 76, pp. 22–35. Springer.

DEKEYSER, I. 1982 Etude d'un jet plan dissymétrique chauffé en régime turbulent incompressible. Thèse Docteur Sciences, Université d'Aix–Marseille II.

FABRIS, G. 1979 Turbulent temperature and thermal flux characteristics in the wake of a cylinder. In *Turbulent Shear Flows I* (ed. F. Durst, B. E. Launder, F. W. Schmidt & J. H. Whitelaw), pp. 55–70. Springer.

GRAD, H. 1949 On the kinetic theory of rarefied gases. *Communs Pure Appl. Maths* **2**, 331–407.

HANJALIC, K. & LAUNDER, B. E. 1972 A Reynolds stress model of turbulence and its application to thin shear flow. *J. Fluid Mech.* **52**, 609–638.

KURBATSKII, A. F. 1975*a* Numerical modelling of turbulent transport processes in the mixing zone. *Zh. Prikl. Mekh. Tehn. Fiz.* **3**, 125–133. [English transl. *J. Appl. Mech. Tech. Phys.* **16** (1976), 416–422.]

KURBATSKII, A. F. 1975*b* Some statistical characteristics of the diffusion of a chemically reactive contaminant in the turbulent mixing zone. *Zh. Prikl. Mekh. Tehn. Fiz.* **6**, 48–58. [English transl. *J. Appl. Mech. Tech. Phys.* **16** (1976), 878–885.]

KURBATSKII, A. F. & ONUFRIEV, A. T. 1979 Modelling of turbulent transport in the wake of a cylinder using equations for the third moments. *Zh. Prikl. Mekh. Tehn. Fiz.* **6**, 99–107.

KURBATSKII, A. F. 1979 On modelling of turbulent convection created by buoyancy. *Inst. of Theor. Appl. Mech. U.S.S.R. Acad. Sci., Siberian Branch, Novosibirsk, Preprint* no. 6.

ORSZAG, S. 1970 Analytical theories of turbulence. *J. Fluid Mech.* **41**, 363–386.

TOWNSEND, A. A. 1956 *The Structure of Turbulent Shear Flow*. Cambridge University Press.

WILLIS, G. E. & DEARDORFF, J. W. 1974 A laboratory model of the unstable planetary boundary layer. *J. Atmos. Sci.* **31**, 1297–1307.